Lens Tolerance for Optics Utilizing Coherent Light

In designing an optical imaging system that utilizes coherent light, it is necessary to estimate the dependence of image quality upon lens quality. In particular, the question arises, is there a different tolerance for optics that utilize coherent light as compared to tolerances for optics utilizing incoherent light? This memo will show that at worst, the two tolerances are equal and, in general, the tolerance for coherent light might be relaxed.

To answer this question, we will first consider one dimensional image formed in:

$$\text{coherent light} \qquad i_c(x) = | t_c(x) * a_c(x) |^2 \qquad (1)$$

$$\text{incoherent light} \qquad i_i(x) = t_i(x) * a_i(x) \qquad (2)$$

where:

$$t_c(x) = \text{spread function in coherent light}$$

$$t_i(x) = | t_c(x) |^2 = \text{spread function in incoherent light} \qquad (3)$$

$$a_c(x) = \text{object complex electric field transmission}$$

$$a_i(x) = | a_c(x) |^2 = \text{object intensity transmission} \qquad (4)$$

$$* \quad \text{means convolution.}$$

Declass Review by NIMA / DoD

STAT

Branch Research

There are two differences of interest in Equations 1 to 4. First, (Eq. 1) coherent imagery is nonlinearly related to the coherent spread function. The relation is linear for incoherent light. Secondly, (Eq. 3) the spread function for incoherent light is non-negative, $t_i(x) > 0$ while the spread function for coherent light may not only be negative but a complex quantity. In this memo we will consider only objects without phase variation i.e., $a_c(x)$ is real and non-negative.

Several authors (1) to (4) consider some effects of aberrations upon coherent imagery. Maréchal (1) considers the effect of aberrations on the central intensity of bright points and lines on a dark field and the reverse case of dark points and lines on a bright field. Welford (2) considers the effect of aberrations on images of bright lines in reference to spectroscopes. Steel considers the effect of aberrations and pupil obscurations on the images of points, lines and edges in reference to a reflecting microscope. These three references treat objects of high contrast. Since coherent imagery is nonlinear it does not follow that these results represent general tolerances for coherent imagery.

Consider the problem of accurately reimaging a continuous tone photographic negative. The results of references 1 to 3 are of limited value since large areas of the negative consist of low contrast objects. This imagery also differs significantly from microscopy because the photographic negative is band limited in frequency. This means that although there is always finer detail to be seen in a microscope image, there is nothing to be found beyond the cut-off frequency of the system that took the original negative. For simplicity we will consider this cut-off frequency to be $w = 1$ radian per unit length.

Coherent imagery is the only way to form an exact replica of a band limited function. Its particular advantage is to preserve or enhance the normally low contrast of small objects. From equations 1 and 2, we find the image of $g(x)$, a low contrast object with no phase variation, is approximately:

$$i_c(x) = 1 + g(x) * \text{Re} \{ t_c(x) \} \text{ for } \begin{cases} g(x) \text{ real} \\ |g(x)| \ll 1 \end{cases} \tag{5}$$

$$i_i(x) = 1 + g(x) * t_i(x) \tag{6}$$

where $\text{Re}\{ \}$ means the real part of $\{ \}$. From Equation 5 we form an approximate spread function and transfer function for real low contrast objects.

The low contrast spread function is:

$$t_a(x) = \text{Re} \{ t_c(x) \} \tag{7}$$

$$= \frac{1}{2} [t_c(x) + t_c^*(x)].$$

For the coherent transfer function:

$$T_c(w) = e^{j\,\theta(w)} \text{ for } |w| < 1 \tag{8}$$

Branch   Research

we form the low contrast transfer function which is the Fourier transform of equation 7

$$T_a(w) = \frac{1}{2} [T_c(w) + T_c^*(-w)] \tag{9}$$

$$= \cos \theta_e (w) e^{j \theta_o (w)}$$

where $\theta_e(w)$ and $\theta_o(w)$ are the even and odd parts respectively of $\theta(w)$ the coherent transfer function phase aberrations.

The corresponding incoherent transfer function is:

$$T_i(w) = T_c(w) * T_c^*(-w) \tag{10}$$

For low contrast coherent imagery the modulus of the transfer function is a function of only the even part of the phase aberrations and the argument only of the odd part of the phase aberrations. This is not so for incoherent imagery where the modulus and arguments of the incoherent transfer function are functions of both the even and odd parts of the phase aberrations except for the special case when the odd part is zero then the argument is also zero.

As an example, consider a square law phase aberration (corresponding to an error in focus) across the aperture of an optical system for which:

$$\theta(w) = 2\pi b w^2 \tag{11}$$

The maximum error is b wavelengths at $w = 1$. From equations 9 and 10, we find:

$$T_a(w) = \cos (2\pi b w^2) \text{ for } |w| < 1 \tag{12}$$

$$T_i(w) = \frac{\sin[2\pi b (2|w|-w^2)]}{4 \pi b |w|} \qquad \text{for } |w| < 2. \tag{13}$$

Equations (12) and (13) are shown in Figure 1 for several values of b. Negative values (180° phase reversal) are indicated by a negative sign. Figure 1 is intended as an illustration only and is not to be interpreted as a tolerance.

In general, $| T_a(w) | > | T_i(w) |$ in the region of interest, $|w| < 1$, even if $T_a(w)$

has small phase aberrations and $T_i(w)$ does not. For this reason, it appears that if a lens performs satisfactorily by conventional standards with incoherent light it should be at least as satisfactory in coherent light. The exact degree of aberration permitted awaits a quantitative measure of image quality. Work is presently in

progress at [   ] to find such a measure.

Branch   Research

The tolerance placed on lens surface polish should be determined by the amount of
scattered light which can be tolerated in the system. The average light scattered
is the same with coherent and incoherent light but the microscopic nature of the
scattered light will differ. The coherently scattered light will exhibit a character-
istic granular structure while the incoherently scattered light will be uniform.
If one considers the light scattered by a bright line image of brightness R into a
dark region due to small random surface roughness, one finds that the dark region
will have an average scattered light background of unity at a given location when:

$$\left[\frac{2\pi\,(n-1)\,\delta\,rms}{\lambda}\right]^2 = R \tag{14}$$

Here δ rms is the root-mean-variation in surface roughness per cycle for all surfaces
and n the refractive index. As an example for a photographic imaging system, we can
take $R = 10^4$ as the largest useable range of intensities because of the limited
dynamic range of photographic film, $n = 1.5$ and $\lambda = 6328°A$ for the helium-neon laser.
For these conditions, one finds roughness for all surfaces:

$$\delta_{rms} < 20\,\overset{o}{A} \quad \text{per cycle for all surfaces} \tag{15}$$

For each of N surfaces:

$$\delta_{rms} < \frac{20}{\sqrt{N}}\,\overset{o}{A} \quad \text{per cycle per surface} \tag{16}$$

if the roughness is random.

Localized defects such as bubbles, digs and scratches present a different problem.
In addition, depending upon the light source, reflection may present a problem.
These will be considered in the next memo.

JK/jf
References:

1. Maréchal, A., Optical Image Evaluation, p. 9-22, National Bureau of Standards
    Circular 526, (1954)

2. Welford, W.T., Aberration Tolerances for Spectra Line Images, Optica Acta,
    Vol. 10, No. 2, p. 121-127, (April 1963)

3. Steel, W. H., Étude des Effets Combines des Aberrations et d'ine Obturation
    Centrale de la Pupille sur le Contraste des Images Optiques, Revue d'Optique
    vol. 32, No. 1, P. 4-26 (1953)

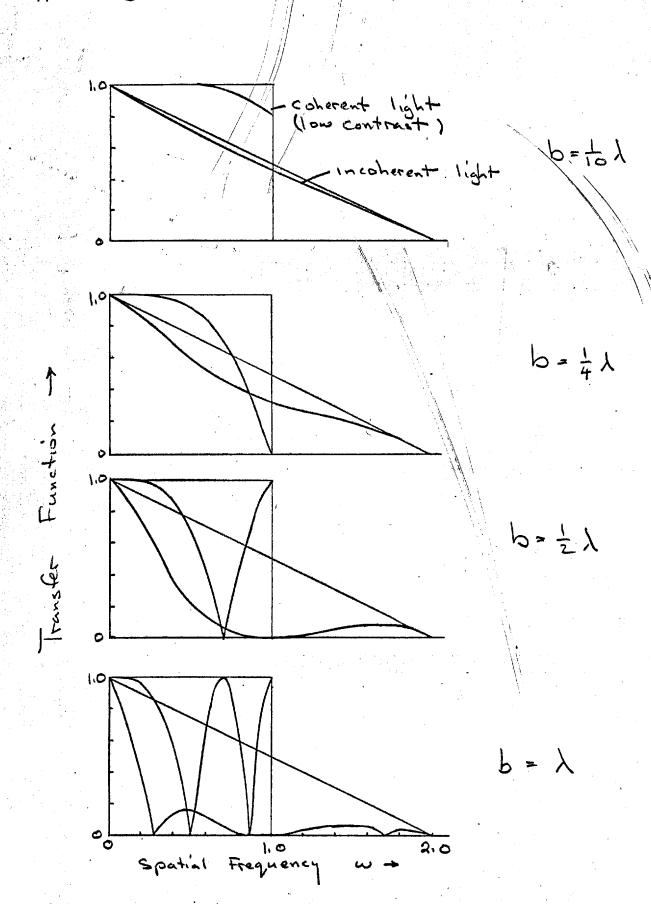4. Born, M., and Wolf, E., Principle of Optics, ch. 9, Pergamon Press, (1959)

Figure 1 : Transfer Functions for Focus Error